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Predictive Feedback Control

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Abstract **− In this work a new method for designing predictive controllers for linear SISO systems is presented. It uses only one prediction of the process output *J* time intervals ahead to compute the correspondent future error. The predictive feedback controller is defined by introducing a filter that weights the last *w*-predicted errors. In this way, the resulting control action is computed by observing the system future behavior and also by weighting present and past errors. This last feature improves the closed-loop performance to disturbance rejection as shown through simulation results.**

1. INTRODUCTION

The use of different kinds of linear models to predict the future behavior of the process output has stimulated the development of a wide group of control. One of the most relevant is the model predictive control (MPC). The predictive control concepts can be thought as an extension of the one–step ahead approach of optimal l_2 control theory, which calls for one–step ahead inversion of the input–output model to produce the control action. This simple inversion approach is not suitable for non–minimum phase system, which will cause the control input to grow unbounded while the controlled output remains bounded. By introducing a proper dynamic into the controller structure, this problem can be overcome.

In this work a new control algorithm is presented. The approach is based on the use of one prediction of the process output J time intervals ahead to compute the corresponding future error. The proposed controller called *predictive feedback*, uses the past predicted errors instead of using plain feedback errors as in classical feedback controllers. Hence, the resulting control action is computed by observing the system future behavior and also by weighting present and past errors. This control strategy combines the predictive capacity, which results in good performance for set-point changes and time-delay systems, with the classical use of feedback information, which improves the closed-loop performance. Depending on the value of J, different control algorithms naturally emerge from the proposed controller. Robust stability and closed-loop performance issues have been analyzed to find some criteria to choose the parameters of the controller.

The paper is organized as follow: in Section 2, the expressions for a general J-step ahead output prediction is presented. In Section 3 the basic formulation of predictive feedback controller design is derived. Besides, the relationship between the proposed controller and other control algorithms is established. The closed-loop stability and performance of the resulting controller are analyzed in Section 4. In Section 5 we show the results obtained from the application of the proposed algorithm to a nonlinear continuous stirred tank reactor. Finally, the conclusions are presented in Section 6.

2. SINGLE-STEP OUTPUT PREDICTOR

Consider a SISO system described by the ARMAX model

$$y(k) = -\sum_{j=1}^{n_y}\tilde{a}_j y(k-j) + \sum_{j=0}^{n_u}\tilde{b}_j u(k-t_d-j) + \sum_{j=0}^{n_u}\tilde{c}_j d_m(k-j) + d(k), \quad (1)$$

where the load disturbance $d_m(k)$ and the non-measurable disturbance $d(k)$ are deterministic. Defining the following variables

$$p \geq max\left(n_y, n_u + t_d, n_d\right), \quad (2)$$

$$\tilde{\alpha}_j = \begin{cases} -\tilde{a}_j & j = 1,2,\ldots,n_y, \\ 0 & j > n_y, \end{cases} \quad (3.a)$$

$$\tilde{\beta}_j = \begin{cases} 0 & j = 1,2,\ldots,t_d, \\ \tilde{b}_j & j = t_d+1,\ldots,t_d+n_y, \\ 0 & j > n_y, \end{cases} \quad (3.b)$$

$$\tilde{\delta}_j = \begin{cases} \tilde{c}_j & j = 1,2,\ldots,n_d, \\ 0 & j > n_d, \end{cases} \quad (3.c)$$

the system output $y(k)$ can be written as follow

$$y(k) = \sum_{j=1}^{p}\tilde{\alpha}_j y(k-j) + \sum_{j=0}^{p}\tilde{\beta}_j \mathrm{u}(k-j) + \sum_{j=0}^{p}\tilde{\delta}_j d_m(k-j) + d(k). \quad (4)$$

By shifting a time step, we obtain the one-step ahead prediction

$$\begin{aligned}\hat{y}(1,k) = {} & \tilde{\alpha}_1 y(0,k) + \sum_{j=1}^{p-1}\tilde{\alpha}_{j+1} y(k-j) + \tilde{\beta}_0 u(k+1) + \sum_{j=0}^{p-1}\tilde{\beta}_{j+1} u(k-j) \\ & + \tilde{\delta}_0 d_m(k+1) + \sum_{j=0}^{p-1}\tilde{\delta}_{j+1} d_m(k-j) + d(k+1),\end{aligned} \quad (5)$$

where the first index stands the number of step ahead samples that the system output is computed (k+1) and the second one is the time when the prediction is computed (k). Replacing (4) into (5) yields

$$\begin{aligned}\hat{y}(1,k) = {} & \sum_{j=1}^{p}\tilde{\alpha}_j^1 y(k-j) + \tilde{\beta}_0 u(k+1) + \sum_{j=0}^{p}\tilde{\beta}_j^1 u(k-j) \\ & + \tilde{\delta}_0 d_m(k+1) + \sum_{j=0}^{p}\tilde{\delta}_j^1 d_m(k-j) + d(k+1),\end{aligned} \quad (6)$$

where the coefficients of the predictor are given

$$\tilde{\alpha}_j^1 = \tilde{\alpha}_{j+1} + \tilde{\alpha}_1\tilde{\alpha}_j; \tilde{\beta}_j^1 = \tilde{\beta}_{j+1} + \tilde{\beta}_1\tilde{\beta}_j; \tilde{\delta}_j^1 = \tilde{\delta}_{j+1} + \tilde{\delta}_1\tilde{\delta}_j \quad j = 1,2,\ldots,p. \quad (7)$$

The system output at time k+1, in absence of the output measurement at time k, can be expressed as a linear combination of past input and output data. In this notation, the current output $\hat{y}(k)$ is given by $\hat{y}(0,k)$ with the coefficients given by $\tilde{\alpha}_j^0 = \tilde{\alpha}_j, \tilde{\beta}_j^0 = \tilde{\beta}_j$ and $\tilde{\delta}_j^0 = \tilde{\delta}_j \quad \forall j$.

Applying the same procedure J times we may express the system output, at time k+J, through

$$\begin{aligned}\hat{y}(J,k) = {} & \sum_{j=0}^{J}\tilde{\beta}_0^{J-j} u(k+j) + \sum_{j=1}^{p}\tilde{\alpha}_j^J y(k-j) + \sum_{j=1}^{p}\tilde{\beta}_j^J u(k-j) \\ & + \sum_{j=0}^{J}\tilde{\delta}_0^J d_m(k+j) + \sum_{j=0}^{J}\tilde{\delta}_j^J d_m(k-j) + d(k+J),\end{aligned} \quad (8)$$

where the coefficients $\tilde{\alpha}_j^J, \tilde{\beta}_j^J$ and $\tilde{\delta}_j^J \; \forall j$ are given by

$$\begin{aligned}\tilde{\alpha}_j^J &= \tilde{\alpha}_{j+1} + \sum_{l=1}^{J}\tilde{\alpha}_1\tilde{\alpha}_j^{J-l} && J \leq p, j = 1,2,\ldots,p, \\ \tilde{\alpha}_j^J &= \sum_{l=1}^{p}\tilde{\alpha}_1\tilde{\alpha}_j^{J-l} && J > p,\end{aligned} \quad (9.a)$$

$$\begin{aligned}\tilde{\beta}_j^J &= \tilde{\beta}_{j+1} + \sum_{l=1}^{J}\tilde{\alpha}_1\tilde{\beta}_j^{J-l} && J \leq p, j = 0,1,\ldots,p, \\ \tilde{\beta}_j^J &= \sum_{l=1}^{p}\tilde{\alpha}_1\tilde{\beta}_j^{J-l} && J > p,\end{aligned} \quad (9.b)$$

$$\tilde{\delta}_j^J = \tilde{\delta}_{j+1} + \sum_{l=1}^{J} \tilde{\alpha}_1 \tilde{\delta}_j^{J-l} \quad J \le p,\ j = 0,1,\ldots,p,$$
$$\tilde{\delta}_j^J = \sum_{l=1}^{p} \tilde{\alpha}_1 \tilde{\delta}_j^{J-l}, \qquad J > p. \tag{9.c}$$

Observation of equations (9) shows that $\tilde{\alpha}_j^J, \tilde{\beta}_j^J$ and $\tilde{\delta}_j^J$ $\forall j$ are a linear combination of its past p parameters weighted by the parameters $\tilde{\alpha}_l$ l=1, 2,…, p, and the coefficients $\tilde{\beta}_j^0$ and $\tilde{\delta}_j^0$ are the j th of the impulse response coefficients ($\tilde{h}_j$ and $\tilde{h}_j^m$) of $\tilde{G}p$ and $\tilde{G}d$ respectively [6]. Defining the predictors

$$\tilde{P}_y\left(J,q^{-1}\right) = \sum_{j=1}^{p} \tilde{\alpha}_j^J q^{-j};\ \tilde{P}_u\left(J,q^{-1}\right) = \sum_{j=1}^{p} \tilde{\beta}_j^J q^{-j};\ \tilde{P}_d^m\left(J,q^{-1}\right) = \sum_{j=1}^{p} \tilde{\delta}_j^J q^{-j}, \tag{10}$$

the J-step ahead prediction (8) can be written as follow

$$\hat{y}(J,k) = \sum_{j=0}^{J} \tilde{h}_{J-j} u(k+j) + \tilde{P}_y\left(J,q^{-1}\right) y(k) + \tilde{P}_u\left(J,q^{-1}\right) u(k)$$
$$+ \sum_{j=0}^{J} \tilde{h}_{J-j}^m d_m(k+j) + \tilde{P}_d^m\left(J,q^{-1}\right) d_m(k) + d(k+J). \tag{11}$$

Since future control actions are unknown, this prediction is not realisable. To turn it realisable a statement must be made about how the input variables are going to move in the future. For example, the simplest rule is to assume that all the inputs will not move in future

$$u(k+j) = u(k+j-1) \quad j = 0,1,\ldots,J,$$
$$d_m(k+j+1) = d_m(k+j), \tag{12}$$
$$d(k+j+1) = d(k+j),$$

which implies that the future changes are equal to zero. Then, the prediction (11) becomes

$$\hat{y}^0(J,k) = \left[\tilde{a}_J q^{-1} + \tilde{P}_u\left(J,q^{-1}\right)\right] u(k) + \tilde{P}_y\left(J,q^{-1}\right) y(k)$$
$$+ \left[\tilde{a}_J^m q^{-1} + \tilde{P}_d^m\left(J,q^{-1}\right)\right] d_m(k) + d(k), \tag{13}$$

where the upper script ^{0} recalls the condition $\Delta u(k+j)=0$ $j=0,1,\ldots,J$ is included and, $\tilde{a}_J$ and $\tilde{a}_J^m$ are the Jth coefficients of step responses of $\tilde{G}p$ and $\tilde{G}d$ respectively. The disturbance term $d(k)$ is computed from the output measurement $y(k)$ and the system model through

$d(k) = y(k) - \hat{y}(0,k).$

This term lumps together possible unmeasured disturbance and inaccuracies, due to plant-model mismatch. Replacing this term in (13) and rearranging we obtain the *corrected open-loop prediction* is given by

$$\hat{y}^0(J,k) = \left[\tilde{a}_J q^{-1} + \mathcal{P}_u\left(J,q^{-1}\right)\right] u(k) + \left[1 + \mathcal{P}_y\left(J,q^{-1}\right)\right] y(k)$$
$$+ \left[\tilde{a}_J^m q^{-1} + \mathcal{P}_d^m\left(J,q^{-1}\right)\right] d_m(k), \tag{14}$$

where

$$\mathcal{P}_y\left(J,q^{-1}\right) = \tilde{P}_y\left(J,q^{-1}\right) - \tilde{P}_y\left(0,q^{-1}\right),\ \mathcal{P}_u\left(J,q^{-1}\right) = \tilde{P}_u\left(J,q^{-1}\right) - \tilde{P}_u\left(0,q^{-1}\right),$$
$$\mathcal{P}_d^m\left(J,q^{-1}\right) = \tilde{P}_d^m\left(J,q^{-1}\right) - \tilde{P}_d^m\left(0,q^{-1}\right). \tag{15}$$

3. PREDICTIVE FEEDBACK CONTROL

The predictive control concepts can be thought as an extension of the one–step ahead approach of optimal l_2 control theory, which calls for one–step ahead inversion of the input–output model to produce the control action]. This simple inversion approach is not suitable for non-minimum phase system, which will cause the control input to grow unbounded while the controlled output remains bounded.

Revising the assumption used to go from (11) to (13), observe that if the control movement $\Delta u(k) \neq 0$, then

$$\hat{y}(J,k) = \hat{y}^0(J,k) + \tilde{a}_J \Delta u(k)\ . \tag{16}$$

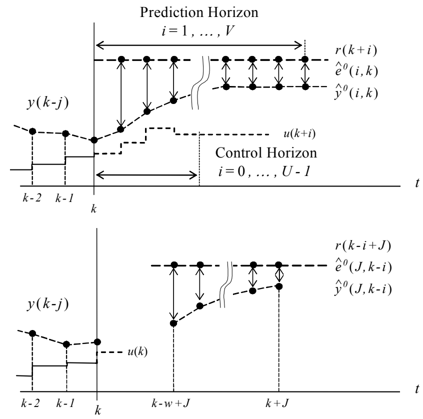

Fig. 1 General MPC and predictive feedback set-ups

Subtracting this prediction from the reference variable $r(k+J)$, we obtain the predicted error

$$\hat{e}(J,k) = \hat{e}^0(J,k) + \tilde{a}_J \Delta u(k)\ . \tag{17}$$

The control action can be computed in the similar way as standard predictive controllers, minimising the following performance measure

$$L(k) = W(q^{-1})\hat{e}^2(J,k) + R(q^{-1})\Delta u^2(k)\ , \tag{18}$$

where $W(q^{-1}) = W_n(q^{-1})/W_d(q^{-1})$ and $R(q^{-1}) = R_n(q^{-1})/R_d(q^{-1})$ are stable weighting functions. Then, the control action that minimises this performance index is given by

$$u(k) = \frac{\tilde{a}_J W_n(q^{-1}) R_d(q^{-1})}{\tilde{a}_J W_n(q^{-1}) R_d(q^{-1}) + \tilde{a}_J W_n(q^{-1}) R_d(q^{-1})}\, \hat{e}^0(J,k)\ , \tag{19}$$

or simply

$$u(k) = \Gamma(q^{-1})/\Phi(q^{-1})\, \hat{e}^0(J,k) = F(q^{-1})\, \hat{e}^0(J,k)\ . \tag{20}$$

Since (19) and (20) uses only one prediction, the delay operator q^{-j} is applied to the time instant at which the prediction is calculated (see figure 1). Hence, the control action $u(k)$ is given by

$$u(k) = \sum_{j=0}^{w} \gamma_j \hat{e}^0(J,k-j) + \sum_{j=1}^{v} \phi_j u(k-j)\ , \tag{21}$$

where $\hat{e}^0(J,k\text{-}j)$ is the open-loop predicted error at time k-j+J based on measurement until time k-j, $u(k\text{-}j)$ is the past control action at time k-j, and γ_j $j=0,1,\ldots,w$ and ϕ_j $j=0,1,\ldots,v$ are the coefficients of the polynomials $\Gamma(z)$ and $\Phi(z)$.

The predictive feedback control law can be derived from the last equation, replacing the open-loop error $\hat{e}^0(J,k–j)$ with their components we obtain the *predictive feedback* control law

$$u(z) = \frac{\Gamma(z) z^J}{\Phi(z) + \Gamma(z)\left[\tilde{a}_J z^{-1} + \mathcal{P}_u(J,z)\right]} r(z) - \frac{\Gamma(z)\left[1 + \mathcal{P}_y(J,z)\right]}{\Phi(z) + \Gamma(z)\left[\tilde{a}_J z^{-1} + \mathcal{P}_u(J,z)\right]} y(z)$$
$$- \frac{\Gamma(z)\left[\tilde{a}_J^m z^{-1} + \mathcal{P}_d^m(J,z)\right]}{\Phi(z) + \Gamma(z)\left[\tilde{a}_J z^{-1} + \mathcal{P}_u(J,z)\right]} d_m(z), \tag{22}$$

whose structure is shown in figure 2. This figure shows a block diagram of how the control action is computed. It is apparent that it uses the plant model to estimate the output at the present time $\hat{y}(0,k)$. This value is then compared with the actual measurement $y(k)$ to detect modelling errors and external disturbances. Then, the prediction of the disturbance $d^0(J,k)$ is added to estimate $d(J,k)$. In other words, given all input changes until the instant k the controller observes the value that would be reached by the system output $\hat{y}^0(J,k–j)$ as if no future

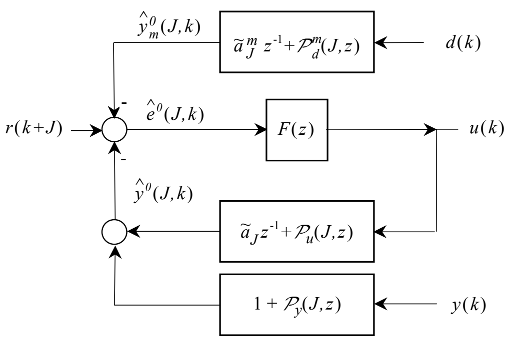

Fig. 2 Structure of the predictive feedback control law

control action is taken, then $u(k)$ is computed such that the performance index (18) is minimised. The parameters of $\Gamma(z)$ and $\Phi(z)$ add additional degrees of freedom to improve the closed-loop performance.

3A.Relationship with other Control Algorithms

The predictive feedback controller consists of a filter, $F(z)$, with the open-loop predictor and the system model in the feedback path. This structure is similar to the Internal Model Control (IMC) parametrization [8]. The only difference is the simultaneous presence of the predictor and the model. The filter could be rewritten using the *Youla parametrization* as

$F(z) = \Gamma(z)/\Phi(z) = Q(z)/1 + Q(z)\tilde{G}p(z)$,

where $Q(z)$ is Youla parameter. In case of stable system, $Q(z)$ is the open-loop controller [8]. So, the predictive feedback controller is a generalisation of the IMC parametrization of the feedback controllers. Depending on the value of the prediction time J and the parameters of the controller, different controllers that have been studied in the specialise literature emerge.

When $J=0$ the open-loop predictor becomes the system model, and the predictive feedback controller becomes the classical feedback controller. For any prediction time greater or equal to the time delay $J \geq \lceil t_d / t_S \rceil$ we can obtain the different predictive controllers.

When $J\,t_S = t_d$ the open-loop predictor is the system model without time delay, and the predictive feedback controller becomes equivalent to a *Smith predictor*.

When $J\,t_S > t_d$ and the parameters of the predictive feedback controller are $w=1$, $\phi_1=-1$, $v=0$ and $\gamma_0=1/a_J$, the resulting controller is the single-prediction controller [5], which is a generalisation of the minimum variance controller. For the particular choice of the prediction time $J = N$, we can derived a family of predictive controllers whose main characteristic is to obtain a closed-loop response that is at least as good as the normalised open-loop response [1][7].

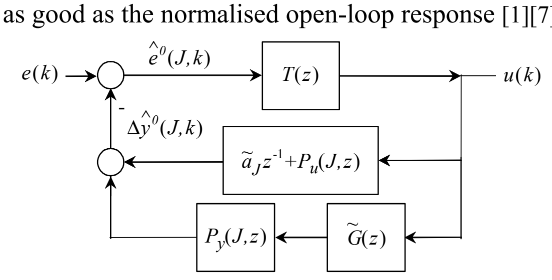

Fig. 3 Structure of the predictive feedback controller

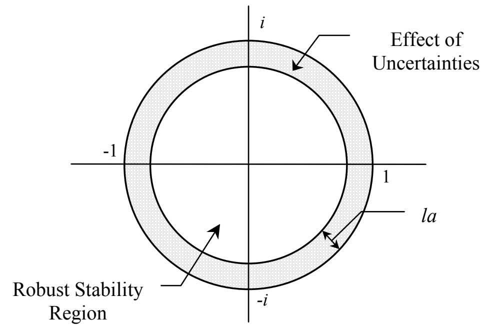

Fig. 4 Geometrical interpretation of equation (24)

ALGORITHM PROPERTIES

Stability Analysis

Now, the stability of the resulting closed-loop is studied. Firstly, we substitute the predictive feedback controller (22) in the characteristic closed-loop equation, and combining with (15), the characteristic equation $T(z)$ can be written as follow

$$\begin{aligned} T(z) = &\left\{\Phi(z) + \Gamma(z)\left[\tilde{a}_J z^{-1} + \tilde{P}_u(J,z) - \tilde{P}_u(0,z) + P_u(0,z)\right]\right\} \\ &+ \left[1 - P_y(0,z)\right] + \Gamma(z) P_u(0,z)\left[\tilde{P}_y(J,z) - \tilde{P}_y(0,z) + P_y(0,z)\right]. \end{aligned} \tag{23}$$

The stability of the closed-loop system depends on both: the prediction time J and the parameters of the filters $\Phi(z)$ and $\Gamma(z)$. So, it may be tested by any traditional stability criteria.

Theorem 1: Given a system controlled by a predictive feedback controller, the closed-loop system will be robustly stable if

$$\frac{1 - \sum_{j=1}^{v} |\phi_j|}{\sum_{j=0}^{w} |\gamma_j|} + \tilde{a}_J > \sum_{j=1}^{p} \left|\tilde{\beta}_j^J\right| + |K_P| \left|\sum_{j=1}^{p} \tilde{\alpha}_j^J\right| + \sum_{j=0}^{p} \left|\beta_j - \tilde{\beta}_j\right| + |K_P| \left|\sum_{j=1}^{p} \left|\alpha_j - \tilde{\alpha}_j\right|\right|. \tag{24}$$

Proof. See Appendix A.

This equation means that the robust stability region} is the circle of radius 1- *la*, so the stability condition (24) guarantees that all of poles of closed-loop system are inside of this region. The terms involved in this condition, ordered from left to right, are: *a*) the contribution of the nominal model, and *b*) the effect of parametric uncertainty. From a geometrical point of view, this condition can be visualised as a reduction of the stability region in a size of *la* (see figure 5)

$$la = \sum_{j=1}^{p} \left|\beta_j - \tilde{\beta}_j\right| + |K_P| \left|\sum_{j=1}^{p} \left|\alpha_j - \tilde{\alpha}_j\right|\right| . \tag{25}$$

Definition 1 Given the polynomial $t(z) = \Sigma_{j=1}^{p} t_j z^{-j}$, the system associate to $t(z)$ is **superstable** if the polynomial $T(z) = 1 + t(z)$ verifies [10]

$$|T(z)| \geq 1 - \|t(z)\|_1 > 0 . \tag{26}$$

The stability criterion (24) guarantees the super stability of the closed-loop system, and they impose a high lower bound for selecting J. Therefore, always there are different prediction times than that one provided by stability condition (24) which leads to stable closed-loop system. They can be found through a direct search in the bounded[1]

$\mathcal{S} = \{J \in \mathcal{N} \wedge J_{Td} \leq J \leq N\}$,

[1] Since we can only use this control law with stable system, the control gain verifies that $K_v = K_{v+1}\ \forall\ v \geq N$, where $N t_S$ is the open loop settling time.

where J_{Td} is the number of samples that represent the time delay t_d.

When $J=1$ the open-loop predictor becomes the system model and the predictive feedback becomes into the classical feedback controller. Under this design criterion, the stability condition (24) is the Dabke condition for system with parametric uncertainties.

In the case of $v=1$ and $\phi_1=-1$, the stability condition (24) becomes

$$\tilde{a}_J > \sum_{j=1}^{p}\left|\tilde{\beta}_j^J\right| + |K_P|\sum_{j=1}^{p}\left|\tilde{\alpha}_j^J\right| + \sum_{j=0}^{p}\left|\beta_j - \tilde{\beta}_j\right| + |K_P|\sum_{j=1}^{p}\left|\alpha_j - \tilde{\alpha}_j\right| . \tag{27}$$

It means that J and the parameters of the filter can be independently selected such that both parameters, J and γ_j $\forall j=0,1,\ldots,w$, independently guarantee the closed-loop stability. This fact means that the prediction time J should be selected like the single-prediction controller, and the filter must be tuned as there is no time delay in the system, because the predictor has compensated it.

Remark 1 The property of the independent selection of the prediction time and parameters of the controllers is held for any controller that verifies

$$\sum_{j=1}^{v}\left|\phi_j\right| = 1, \tag{28}$$

and the equation (24) becomes (27).

Under this condition, J can be varied such that the closed-loop performance is improved. Varying J we modify the closed-loop settling time, accelerating or de-accelerating the system response. So, if we have to control a non-linear system we can choose a different J for each operating region such that we obtain a similar closed-loop response for each one of them. Then, during the operation, we vary J according with the operating region controlled at each sample.

Remark 2 Given a controller that simultaneously satisfy (24) and (28), the closed-loop response of the time-varying system that results from varying the prediction time J has bounded responses.

Proof. See lemma 2 [10].

Performance analysis

The discrete-time superstable systems enjoy numerous important properties. The main one is that they admit simple non-asymptotic estimates for arbitrary initials conditions.

Lemma 1 Given the closed-loop system described by

$$y(k) = -\sum_{j=1}^{m} p_j y(k-j) + \sum_{j=1}^{m} n_j w(k-j) , \tag{29}$$

with initial conditions $|y(-j)| \le \mu$ j=1,2,…,m and bounded disturbances $|w(k)| \le 1$ $\forall k$. Then, the closed-loop system responses is bounded by

$$|y(k)| \le \eta + \|p\|_1^{+1} \max\{0, \mu - \eta\} \quad \forall k \ge 0 , \tag{30}$$

where $\eta = \|p\|_1 / 1 - \|p\|_1$.

Proof. See lemma 1 [10]

Hence, the output of system controlled by a predictive feedback controller that satisfies the stability criteria (24) or (27) and (28) can be estimated for all time steps, not only its asymptotic values. Moreover, for any $c>1$ a k_0 can be found such that $|y(k)| \le c\eta$ $\forall k > k_0$.

In contrast, for stable systems we can guarantee the asymptotic estimates for the output, while the effect of non-zero initial conditions may be very large. These results can be easily extended to time-varying system by analysing the behaviour of the frozen systems.

Finally, we give a heuristic analysis of the closed-loop performance achieved by the generalised predictive feedback controller and then, we compare it with those one provided by a classical feedback controller and a standard MPC controller. To carry out this analysis the resulting control action can be compared for all controllers.

The control actions generated by the predictive feedback control law (21) are obtained by replacing the open-loop error $\hat{e}^0(J,k\text{-}j)$, the result is

$$\begin{aligned} u(k) &= \sum_{j=0}^{w}\gamma_j e(k-j) - \sum_{j=0}^{v}\phi_j e(k-j) \\ &\quad + \sum_{j=0}^{w}\gamma_j\left[\mathcal{P}_u(J,q^{-1}) + \mathcal{P}_y(J,q^{-1})\tilde{G}p(q^{-1})\right]u(k-j), \end{aligned} \tag{31}$$

Note that the two first terms of this equation, ordered from left, are the time implementation of a discrete controller, while the last one is a weighing contribution of the future open-loop deviations at time

$$\Delta\hat{y}^0(J,k-j) = \hat{y}^0(J,k-j) - \tilde{y}(k-j) \quad j = 0,1,\ldots,w \ .$$

They only depend on the past control actions and the system model, therefore the last term of (31) states the effect of the past control actions on the future behaviour of the system. These facts imply that it has significative influence on the closed-loop performance when we have to track setpoint changes, but a negligible one when we have to reject a disturbance because this term has little information about it. So, the two first terms of (31) command the system behaviour during the disturbance rejection, because the measured errors have all the information the information at time k.

Comparing the control action generated by a predictive feedback controller with that produced by a classical feedback controller of the same orders

$$u(k) = \sum_{j=0}^{w}\gamma_j e(k-j) - \sum_{j=0}^{v}\phi_j e(k-j) , \tag{32}$$

we can see that they only differ in the last term: the contributions of the future open-loop deviations $\Delta\hat{y}^0(J,k\text{-}j)$. So, the predictive feedback controller will have a better closed-loop performance for setpoint tracking, especially when the system has a large time delay. However, both controllers will have a similar performance in disturbance rejection task.

Now, we compare the performance achieved by a predictive feedback with that one obtained by a standard MPC controller. The control action generated by a MPC controller is [3]

$$\begin{aligned} u(k) &= \left\{\sum_{j=0}^{V}k_j\right\}e(k) - u(k-1) \\ &\quad + \sum_{j=0}^{V}k_j\left[\mathcal{P}_u(J,q^{-1}) + \mathcal{P}_y(J,q^{-1})\tilde{G}p(q^{-1})\right]u(k-1), \end{aligned} \tag{33}$$

where V is the prediction horizon and k_j $j=1,2,\ldots,V$ is the jth element of the gain vector. In this equation we can see that MPC controllers only use the last measured error and the two first terms, ordered from the left, are a discrete PI controller. Like the predictive feedback controller, the last term is a weighing contribution of the future open-loop deviations at time $(k+j)t_S$.

From equations (31) and (33) we can see that both predictive controllers have a similar structure, they only differ in the number of terms employed by each one. However, we can also see that the predictive feedback controller uses more feedback information than MPC controllers to compute the control actions. Therefore, the predictive feedback controller reduces the effect of disturbances more aggressively than any standard MPC controller, and has better performance than MPC, especially for disturbance rejection problem.

SIMULATIONS AND RESULTS

Consider the problem of controlling a continuously stirred tank reactor (CSTR) in which an irreversible exothermic reaction is carried out at constant volume. This is a nonlinear system originally used by [9] for testing predictive controls algorithms. The objective is controlling the output concentration $Ca(t)$ using the coolant flow rate $q_C(t)$ as the manipulated variable. The output concentration has a measured time delay of $t_d = 0.5$ min

The nonlinear nature of the system is showed in figures 5. Four continuous linear models are determined from the composition responses showed in this figure using subspace identification technique [11]. Notice that these changes imply three different operating points corresponding to the following stationary manipulated flow-rates: 100 $lt\,min^{-1}$, 110 $lt\,min^{-1}$, and 90 $lt\,min^{-1}$.

The controller must be able to follow the reference, so we need to guarantee its controllability in the whole operational region. Hence, assuming a hard constraint is physically used on the coolant flow rate at 110 $lt\,min^{-1}$, an additional restriction for the more sensitive model (model 1) must be considered for the deviation variable

$$u_1(k) \le 10 \,. \tag{34}$$

Besides, a zero-offset steady-state response is demanded, and then the following constraints must be included

$$\textstyle\sum_{j=1}^{v} \phi_j = -1 \,. \tag{35}$$

This assumes that the nominal absolute value for the manipulated is around 110 $lt\,min^{-1}$ and that the operation is kept inside the polytope whose vertices are defined by the linear models. The constraints (34) and (35) are then included in tuning problem.

Now, we define the parameters of the predictive feedback controller to tune the filter parameters using the method proposed by [4]. The orders of the controller's polynomials are arbitrarily adopted such that the resulting controllers include the predictive version of popular PID controller ($v = 1$ and $w = 2$). The predictor of the controller is built using the model corresponding to the more sensitive region (model 1).

Since we demand an integral action in the controller, the prediction time J and the parameters of the controller can be independently fixed (separability condition (28)). So, the prediction time J was chosen such that we obtain the better closed-loop performance in each operational region. The resultant prediction times are summarised in Table 1. During the operation of the system the predictive horizon J is adjusted at each sample according with the operational point, which is defined by the final values of the reference.

Notice in this case that it is the polytope that must be shaped along the time being considered. Hence, the objective function necessary for driving the adjustment must consider all the linear models simultaneously. At a given time instant and operating point, there is no clear information about which model is the convenient one for representing the process. This is because it depends not only on the operating point but also on which direction the manipulated variable is going to move. The simpler way to solve this problem is by proposing the following objective function

$$L(k) = \textstyle\sum_{l=1}^{M} \sum_{k=0}^{N} \theta_l \left[e_l^2(k) + \lambda_l u_l^2(k) \right], \tag{36}$$

where the time span is defined by $N = 200$ and $M = 4$. The control weight λ_l was fixed in a value such that the control energy has a similar effect than errors in the tuning process ($\lambda_l = 0.01$). Since in this application we found no reason to differentiate the models, we adopt $\theta_l = 1 \; \forall \; l = 1,2,3,4$

The problem described to this point has a rapid numerical solution using an algorithm based on sequential quadratic program method. The parameters obtained are the following

$$\gamma_0 = 0.1655; \gamma_1 = -0.1553; \gamma_2 = 0.1043; \phi_1 = -1 \,. \tag{37}$$

In [9], the authors have previously worked with this reactor model for testing different alternatives of predictive controllers. They confronted the results with the responses obtained using a PI controller whose parameters were adjusted by the ITAE criterion; thus, we used the same settings: the gain value, 52 $lt^2 mol^{-1} min^{-1}$ and the integration time constant, 0.46 *min*. The simulation test is also similar to Morningred's work and consists of a sequence of step changes in the reference value.
Figure 6 shows the results obtained when comparing the discrete controller with the mentioned PI. The set point was changed in intervals of 10 min. from 0.09 to 0.125, returns to 0.09, then steps to 0.055 and returns to 0.09. The superior performance of the predictive feedback controller is obtained through a vigorous initial movement in the manipulated variable, which however does not overcome the 110 $lt\,min^{-1}$ limit as shown in Figure 7, but shows more movements than the PI. This fact happens due to the open-loop dynamic of the reactor.

Table 1: Prediction time J for each model

	Model 1	Model 2	Model 3	Model 4
J	12	11	10	11

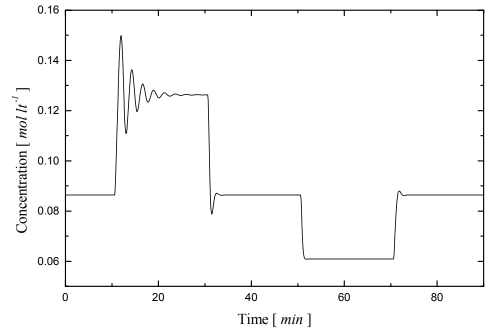

Fig. 5 Open-loop responses to step changes in coolant flow rate

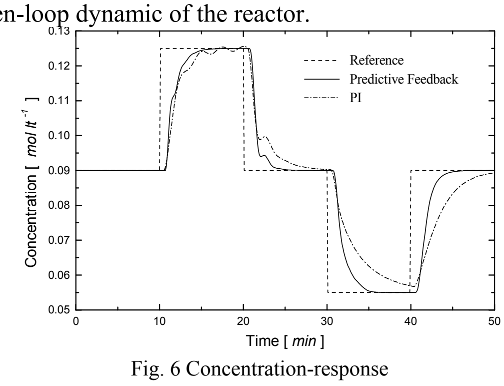

Fig. 6 Concentration-response

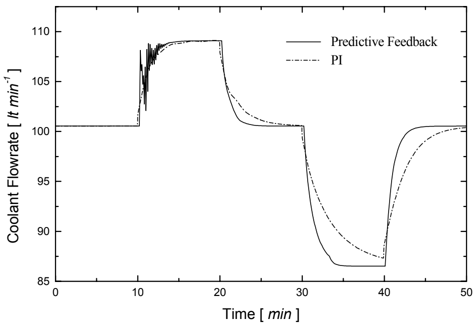

Fig. 7 Manipulated movements

The problem can be solved increasing the control weight λ in the objective function (36) or varying the closed-loop settling time through a greater prediction time J (the other option is easier to implement because we do not need to retune the parameters of the controller). These facts have the same effect: reduce the control energy by degrading the closed-loop performance.

CONCLUSIONS

A new method of designing discrete controllers, for linear SISO systems, has been presented in this work. It uses only one prediction of the system output J time intervals ahead to compute the corresponding future error. Then, the predictive feedback controller is defined by introducing a filter that weights the last predicted errors. In this way, the resulting control action is computed by observing the system future behaviour and the present and past errors. These features enable the predictive feedback controller to combine the capacity of predictive control algorithm, for good setpoint tracking and time delay compensation, with the classical use of the feedback information to improve disturbance rejection.

The character of this controller is governed by one parameter: the prediction time, which defines the closed-loop settling time. Some simple criteria for its selection are provided: they guarantee the robust stability of the closed-loop system. The controller can use the parameters of the filter to enhance the closed-loop performance and to shape the closed-loop time-domain response.

IV. ACKNOWLEDGMENTS

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APPENDIX A

The characteristic closed-loop equation $T(z)$ for the predictive feedback control (22) is given by

$$\begin{aligned} T(z) = D(z)\{\Phi(z) + \Gamma(z)[\tilde{a}_J z^{-1} + \mathcal{P}_u(J,z) + P_u(0,z)]\} \\ + D(z)\Gamma(z)[\tilde{a}_J z^{-1} + \mathcal{P}_y(J,z) + P_y(0,z)]. \end{aligned} \tag{38}$$

Recalling

$D(z) = 1 - P_y(0,z);\ N(z) = P_u(0,z)$,

and combining this expression with (15), the characteristic equation becomes

$$\begin{aligned} T(z) = \{\Phi(z) + \Gamma(z)[\tilde{a}_J z^{-1} + \tilde{P}_u(J,z) - \tilde{P}_u(0,z) + P_u(0,z)]\} \\ + [1 - P_y(0,z)] + \Gamma(z)P_u(0,z)[\tilde{P}_y(J,z) - \tilde{P}_y(0,z) + P_y(0,z)]. \end{aligned} \tag{39}$$

The stability of the closed-loop system depends on the prediction time J and may be tested by any usual stability criteria. First, the following lemma is introduced.

Lemma 2 If the polynomial $T(z)=t_0+\Sigma_{i=1}^{p} t_i z^{-i}$ has the property

$$\inf_{|z|>1} |T(z)| > 0 \ ,$$

then the related closed-loop system will be asymptotically stable [2]. Hence, applying the lemma 1 to (39) results

$$\frac{\Phi(1)}{\Gamma(1)} + \tilde{a}_J > |\tilde{P}_u(J,1)| + |K_P||\tilde{P}_y(J,1)| + |P_u(0,1) - \tilde{P}_u(0,1)| + |K_P||P_y(0,1) - \tilde{P}_y(0,1)| \ , \tag{40}$$

where K_P is the system gain given by $K_P = P_u(0,1)/1 - P_y(0,1)$.

Finally, combining this expression with (10) the stability condition for the predictive feedback controller is obtained

$$\frac{1 - \sum_{j=1}^{v}|\phi_j|}{\sum_{j=0}^{w}|\gamma_j|} + \tilde{a}_J > \sum_{j=1}^{p}|\tilde{\beta}_j^J| + |K_P|\sum_{j=1}^{p}|\tilde{\alpha}_j^J| + \sum_{j=0}^{p}|\beta_j - \tilde{\beta}_j| + |K_P|\sum_{j=1}^{p}|\alpha_j - \tilde{\alpha}_j| \cdot \tag{41}$$