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Robust Adaptive Control using Multiple-Models, Switching and Tuning

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An universally accepted solution to the adaptive control problem has been an elusive search. In spite of several decades of research an accepted design methodology for reliable adaptive control, based on sound theoretical issues and suitable for real implementation, has not been found yet. A common problem in many adaptive control approaches is the instability originated by the presence of uncertainties that arise from different causes:

- **Interactions between the identification and controller design processes**: when adaptation and plant dynamics time scales are similar, the closed-loop system behavior can result in chaos [1], [2] and instability [3].
- **Abrupt controller changes:** the modification of the controller can introduce new behavior in the overall system that is not present in any of the composite subsystems.
- **Poor estimates of parameters:** the presence of non-measurable disturbances and noises lead to poor estimates of parameters and consequently system instabilities [4].

It is clear that there is a need for adaptive control algorithms with the attributes of non-adaptive robust feedback systems: robust stability and robust performance.

Traditional multiple-models switching and tuning is an adaptive control strategy based on the idea of describing the dynamics of the system, using different models for different operating regimes and devise a suitable strategy to find the model that is closest (in behavioral sense) to the current plant dynamics in order to select the appropriate controller [5], [6]. The adaptive control approach proposed in this work is based on the assumption that a set of m *LTI* models Σ_l $l=1,\ldots,m$ ($\boldsymbol{W}$) can approximate a dynamical system in a bounded domain $\boldsymbol{D}$ with accuracy ε [7]. Then, a *LTV* model, which is employed to design the control law, is built from $\boldsymbol{W}$ using switching techniques at every sample. In this way, the *LTV* model will be defined by a set of the closest models *LTI* models to the current dynamic

$$\boldsymbol{M}(k)=\left\{\Sigma_l \,/\, I_l(k)\le I_{\min}(k)+\delta(k)\ \ \delta(k)>0\right\}\subseteq \boldsymbol{W}, \qquad (1)$$

which characterizes the system dynamic at each sample. In this adaptive scheme the monitor signals $I_l(k)$ are employed to decide which model is active during the design stage and $I_{\min}(k)=\min_{l\in[1,m]}\left\{I_l(k)\right\}$. They are generated from the estimation error $e_l(k)=y_l(k)-y(k)$. The parameter $\delta(k)$ is employed to describe the region of $\boldsymbol{D}$ that will be used to design the control law, defining the robustness of the closed-loop system (see Figure 1). In the first few samples several models can have similar behavior and it is difficult to distinguish between them. Therefore, $\boldsymbol{M}(k)$ will include more models than those ones required to represent the system, providing robustness according to the information available at each sampling time. As the time goes by, the indexes $I_l(k)$ will clearly differentiate and $\boldsymbol{M}(k)$ will reduce its size until only the models required to represent the system with the desired accuracy $\delta(k)=\varepsilon$ are selected (Figure 2). At this stage, $\boldsymbol{M}(k)$ will include, at least, the two closest models to the current dynamic. Consequently, the control law designed with $\boldsymbol{M}(k)$ will stabilize the actual system's operating region, and their neighborhood. Following the behavior of $\boldsymbol{M}(k)$, as the time advances the control law will change as the stabilization region is moving through $\boldsymbol{D}$, until the steady-state is achieved. The behavior of $\boldsymbol{M}(k)$ is equivalent to a dynamical partition of the operating domain [8], [9] being carried out during the model selection trough the switching rule.

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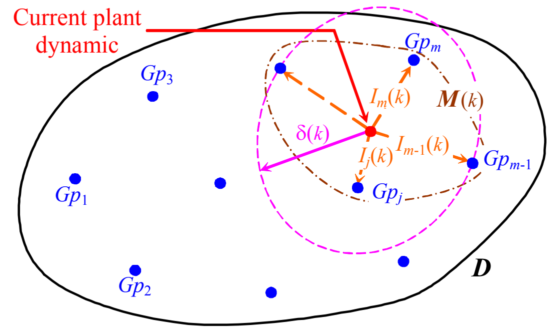

Fig. 1. Geometrical interpretation of set definition (1).

The set $\boldsymbol{M}(k)$ is employed to design the control law $K(k)$ using the following multi-objective optimization problem

$$\min_{K(k)} J\left(K(k),\mathbf{S}(k)\right) \qquad (2.a)$$

$$S_l(k)\left\|A_l+B_lK(k)\right\|\le\sigma_l \quad l=1,\ldots,m,\sigma_l\in[0,1) \qquad (2.b)$$

where the optimization variable $K(k)$ is the controller gain associated to time instant k, $\mathbf{S}(k)=[S_1(k),\ldots, S_m(k)]$ is the vector of switching variables and $J(K,\boldsymbol{S})$ is a general index that measure the closed-loop performance. The switching is

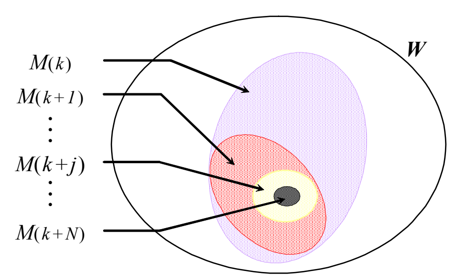

Fig. 2. Time evolution of model subset $\boldsymbol{M}(k)$.

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performed in the objective function $J(K,\boldsymbol{S})$ and in the constraints of the optimization problem (2). This optimization problem is convex in the control law's parameters, and allows the inclusion of additional constraints on the system states and the controller structure. It turn up that the objective function $J(K,\boldsymbol{S})$ is a general index of the form

$$J\left(K,\mathbf{S}\right)=\tfrac{1}{1-\sigma_{\min}(k)}\sum_{l=1}^{m}S_l(k)\left(1+\alpha_l\left\|K(k)\right\|_1\right)\quad \alpha_l\ge 0,\\ \sigma_{\min}(k)=\min_{l\in[1,m]}\{\sigma_l\}. \tag{3}$$

The performance index (3) corresponds to the *Linear Linear Regulator* problem, which arises in control applications [10] and in some specific problems in l_1–optimization [11]. The variables $S_l(k)$ are used to switch-in the appropriate component of the index $J(K,\boldsymbol{S})$ and are obtained by comparing the monitor signals

$$S_l(k)=\mathcal{H}_\delta\left(I_l(k)-I_{\min}(k)\right),\\ \mathcal{H}_\delta\left(x\right)=\begin{cases}1 & x\le\delta \quad \delta\ge 0,\\ 0 & x>\delta.\end{cases} \tag{4}$$

Therefore, the controller gain $K(k)$ is designed employing the closest models to the current plant dynamic. These models are used to measure the closed-loop performance and evaluate the system's constraints. In the mean time, the superstability [12] of the models included in $\boldsymbol{M}(k)$ is guaranteed through constraints (2.*b*), such that the stability of the closed-loop system for any switching sequence is ensured [13].

The structure of the proposed robust adaptive controller is shown in Figure 3. The estimators, monitoring signals generation and switching logic block generates the vector of switching variables $\mathbf{S}(k)$ independently of the controller design. Then, at every sample, the control law is designed by solving the optimization problem (2).

The approach to adaptive control described up to now is equivalent to have an infinite number of controllers in the standard switching multiple-model adaptive scheme [6], with the additional benefit of constraint handling. Implementing these ideas lead to two possible robust adaptive control algorithms:

1. A robust adaptive control algorithm that employs the **same set of models** ($\boldsymbol{M}(k)$) to measure the performance and stabilize the system. Implying the use of one set of switching variables $S_l^S(k)$ for the stabilization and the performance measure ($\mathbf{S}(k)=[\,S_l^S(k)]$) with the objective function

$$J_S\left(K,\mathbf{S}\right)=\sum_{l=1}^{m}S_l^S(k)\left(1+\alpha_l\left\|K(k)\right\|_1\right),\\ \sigma_l\ge S_l^S(k)\left\|A_l+B_lK(k)\right\|_1 \qquad l=1,\ldots,m,\\ S_l^S(k)=H_{\delta(k)}\left(I_l(k)-I_{\min}(k)\right). \tag{5}$$

2. A robust adaptive control algorithm that employees **two different sets**: the subset $\boldsymbol{M}(k)$ to **stabilize** the system and the **closest model** ($I_j(k)=I_{\min}(k)$) to measure the performance. It follows that, the adaptive control algorithm uses the set of switching variables $S_l^S(k)$ for the stabilization and another set $S_l^P(k)$ for the performance evaluation ($\mathbf{S}(k)=[\,S_l^S(k),\,S_l^P(k)]$) with the objective function

$$J_P\left(K,\mathbf{S}\right)=\sum_{l=1}^{m}S_l^P(k)S_l(k)\left(1+\alpha_l\left\|K(k)\right\|_1\right),\\ \sigma_l\ge S_l^S(k)\left\|A_l+B_lK(k)\right\|_1 \qquad l=1,\ldots,m,\\ S_l^P(k)=H_0\left(I_l(k)-I_{\min}(k)\right). \tag{6}$$

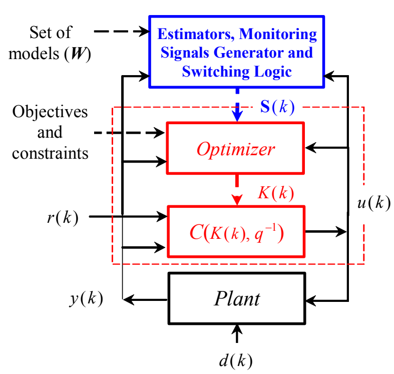

Fig. 3. Structure of the proposed adaptive controller.

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References

[1] I. Mareels and R. Bitmead, “Non-linear dynamics in adaptive control: Periodic and chaotic stabilization”, *Automatica*, vol. 22, no. 6, pp. 641-655, 1986.
[2] I. Mareels and R. Bitmead, “Non-linear dynamics in adaptive control: Periodic and chaotic stabilization - II Analysis”, *Automatica*, vol. 24, no. 4, pp. 485-497, 1988.
[3] B. Anderson, “Failures of adaptive control theory and their resolution”, *Communications in Information and Systems*, vol. 5, no. 1, pp. 1-20, 2005.
[4] B. Anderson, “Adaptive systems, Lack of Persistency of Excitation and Bursting Phenomena”, *Automatica*, vol. 21, no. 3, pp. 247-258, 1985.
[5] A. Morse, “Supervisory control of families of linear set-point controllers - part 1: Exact matching”, *IEEE Trans. on Automatic Control*, vol. 41, no. 10, pp. 1413-1431, 1996.
[6] B. Anderson, T. Brinsmead, F. de Bruyne, J. Hespanha, D. Liberzon and A. Morse, “Multiple model adaptive control I: finite controller covering”, *International Journal of Robust and Nonlinear Control*, vol. 10, no. 12, pp. 909-929, 2000.
[7] G. Angelis, *System analysis, modelling and control with polytopic linear models*, Ph.D. Thesis, University of Eindhoven, 2001.
[8] M. Johansson and A. Rantzer, “Computation of piecewise quadratic Lyapunov function for hybrid system”, *IEEE Transactions on Automatic Control*, vol. 43, no. 4, pp. 555-559, 1998.
[9] A. Rantzer and M. Johansson, “Piecewise linear quadratic optimal control”, *IEEE Transactions on Automatic Control*, vol. 45, no. 4, pp. 629-636, 2000.
[10] K. Ogata, Modern Control Engineering, Upper Saddler River: Prentice Hall, 1990.
[11] J. Yu and A. Sideris, “Optimal induced l_1–norm state feedback control”, in Proc. *36 IEEE Control and Decision Conf.*, San Diego, 1997, pp. 1558-1563.
[12] B. Polyak and M. Halpern, “Optimal design for discrete-time linear systems via new performance index”, *Adaptive Control and Signal Processing*, vol. 15, no. 2, pp. 153-168, 2001.
[13] L. Giovanini and M Benosman, “Adaptive control using multiple models, switching and tuning”, Industrial Control Centre, University of Strathclyde, Tech. Rep. ICC225, Aug. 2005.